Dual-spacecraft reconstruction of a 3-D magnetic flux rope at Earth's magnetopause

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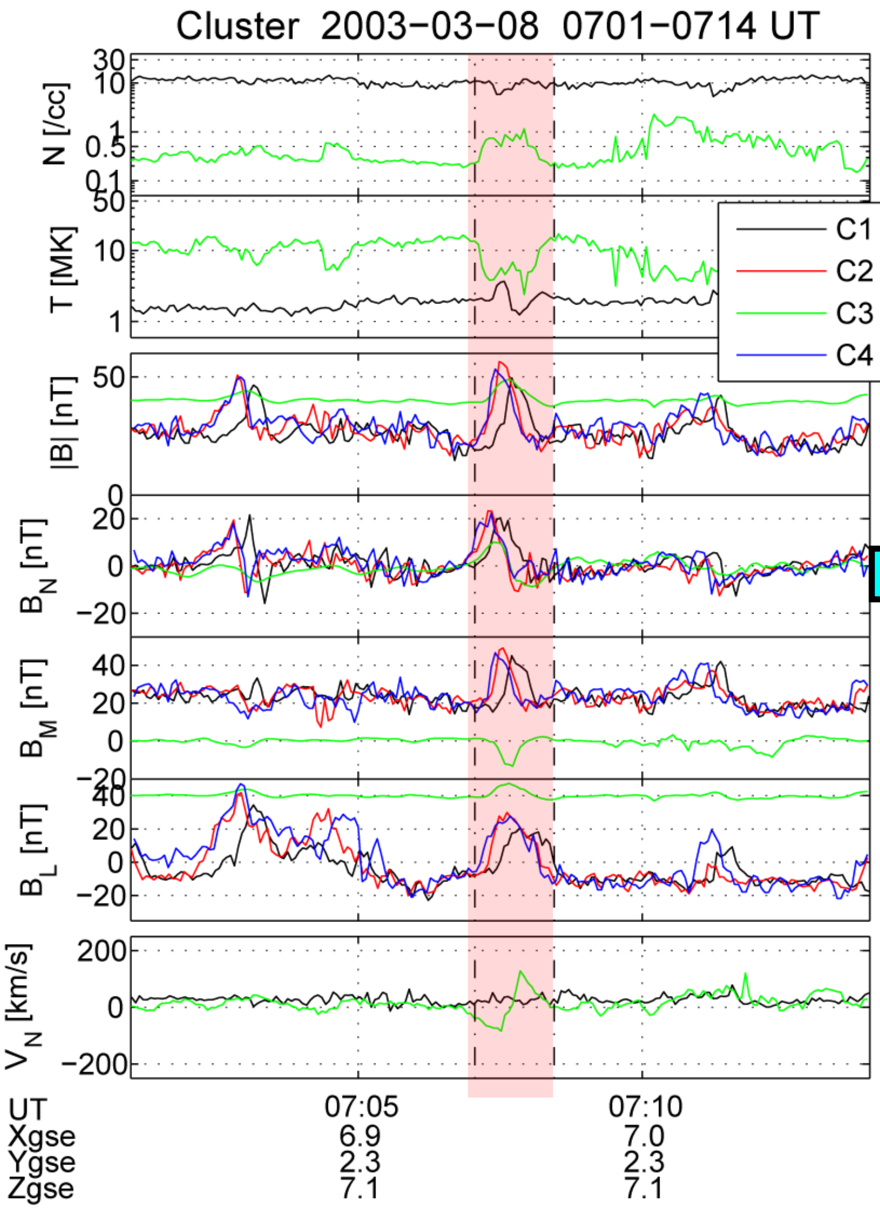
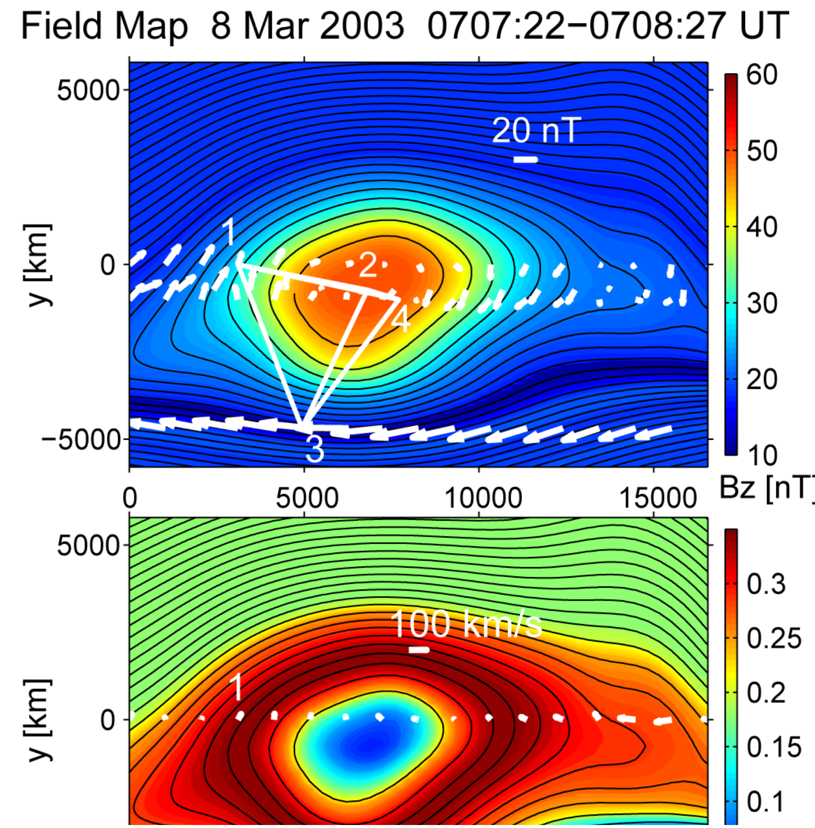
Geospace revisited: a Cluster/MAARBLE/Van Allen Probes Conference

Island of Rhodes, Greece (15-20 September 2014)

Time series data to 2D image





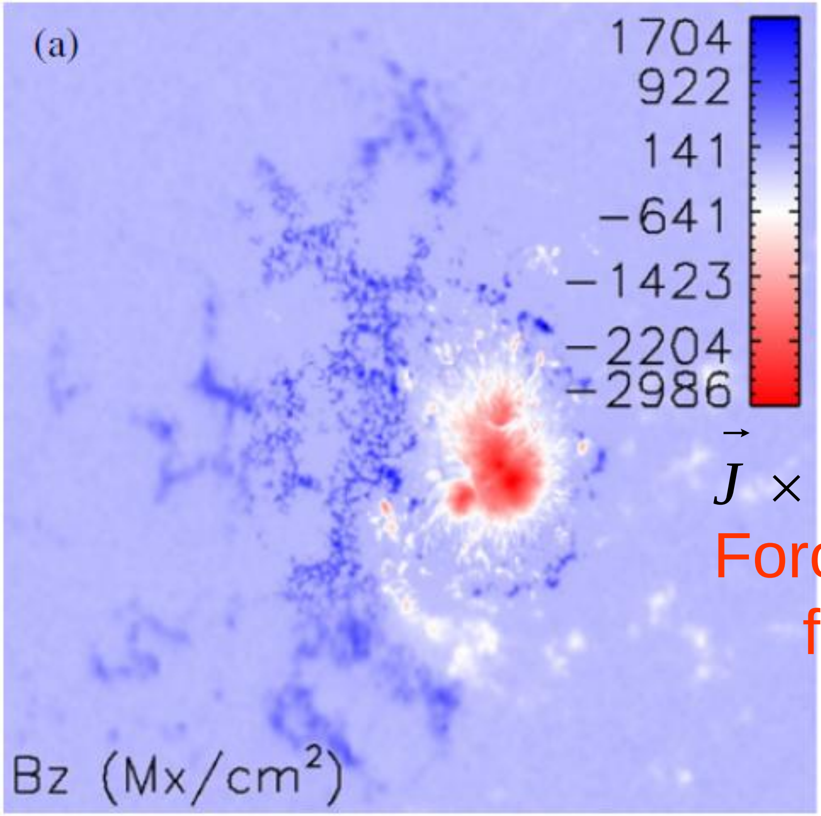
Grad-Shafranov equation

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu_0 \frac{d P_t}{d A} = -\mu_0 j_z(A)$$

Sonnerup+ (GRL04)

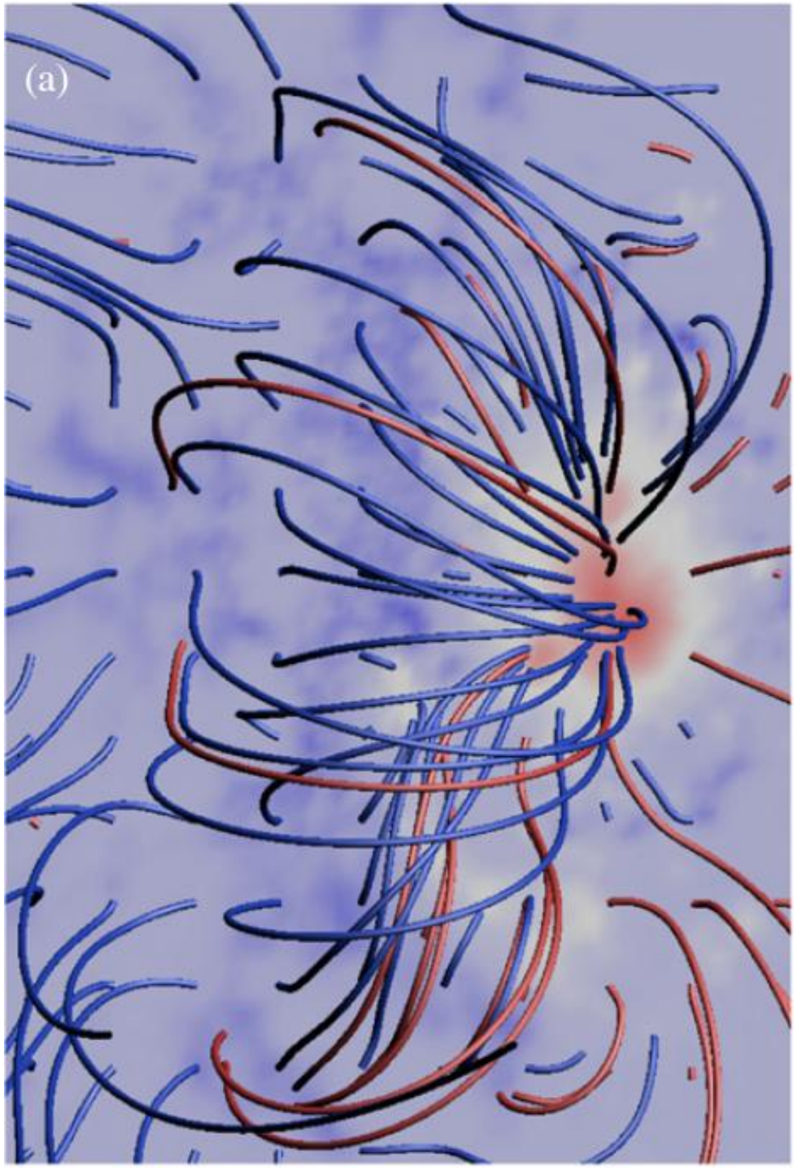
2-D data to 3-D structure

Reconstruction of coronal 3-D fields from photospheric 2-D field measurements



$$\vec{J} \times \vec{B} = 0$$

Force-free field



Wheatland & Leka (ApJ11)
Wiegelmann & Sakurai (LRSP12)

Background

- Sonnerup & Hasegawa (JGR11) developed a technique for reconstructing steady 3-D magnetic field from 2-spacecraft (Sc) data.
- MMS (NASA 4-Sc mission for understanding reconnection) will be launched early 2015.

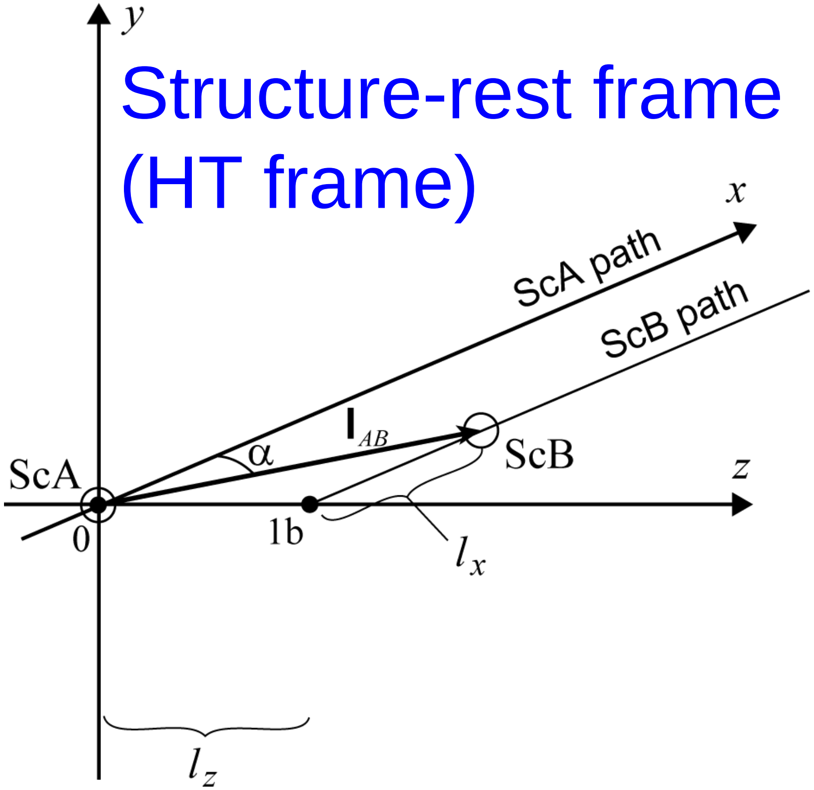
Objective

- To test the 3-D reconstruction technique by applying it to actual Sc data for the first time.
- To extract information on the field/plasma structure for better data interpretation.

Assumptions of the 3D reconstruction

- Time independent
  - Time series of data can be converted to spatial information along the *x* axis.
- Magneto-hydrostatic
  - Flow speed has to be slow in the structure-rest frame.

Structure-rest frame (HT frame)



x axis is parallel to Sc path (anti-|| to V_{HT})

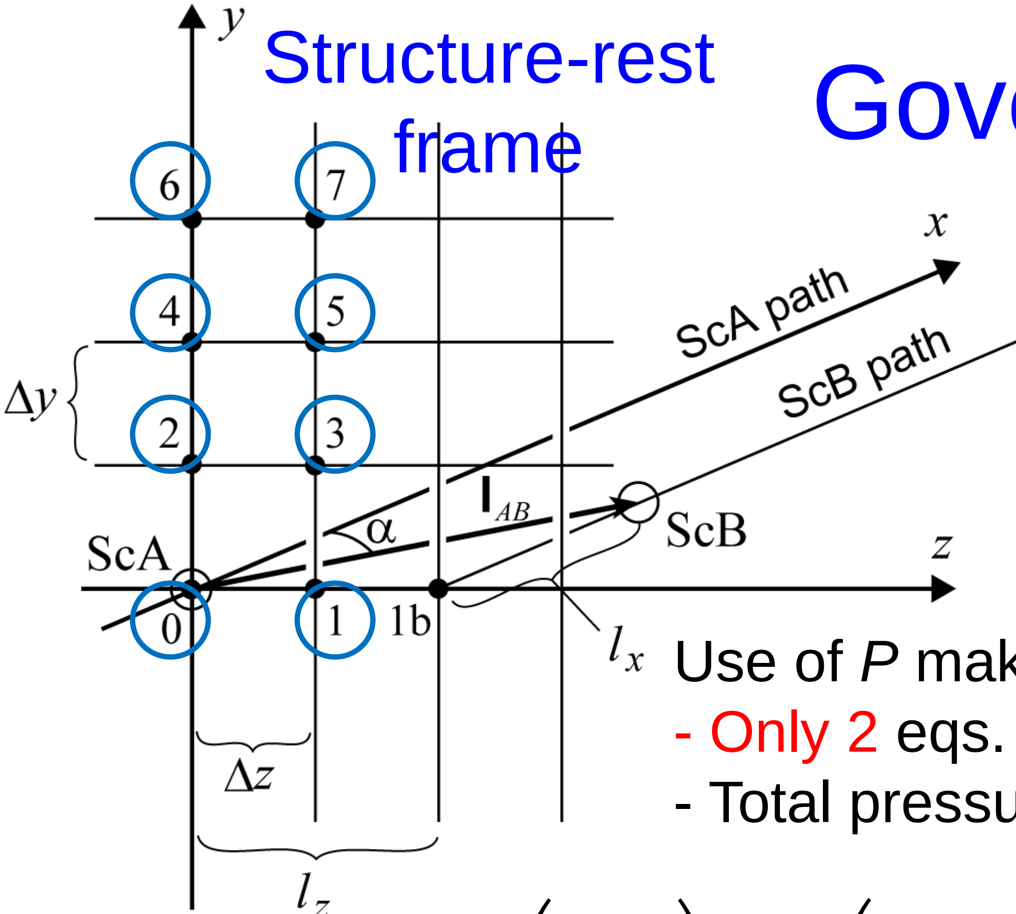
Sonnerup & Hasegawa (JGR11)

Requirement

- ScA & ScB are separated perpendicularly to *x* (along *z*).

Governing equations

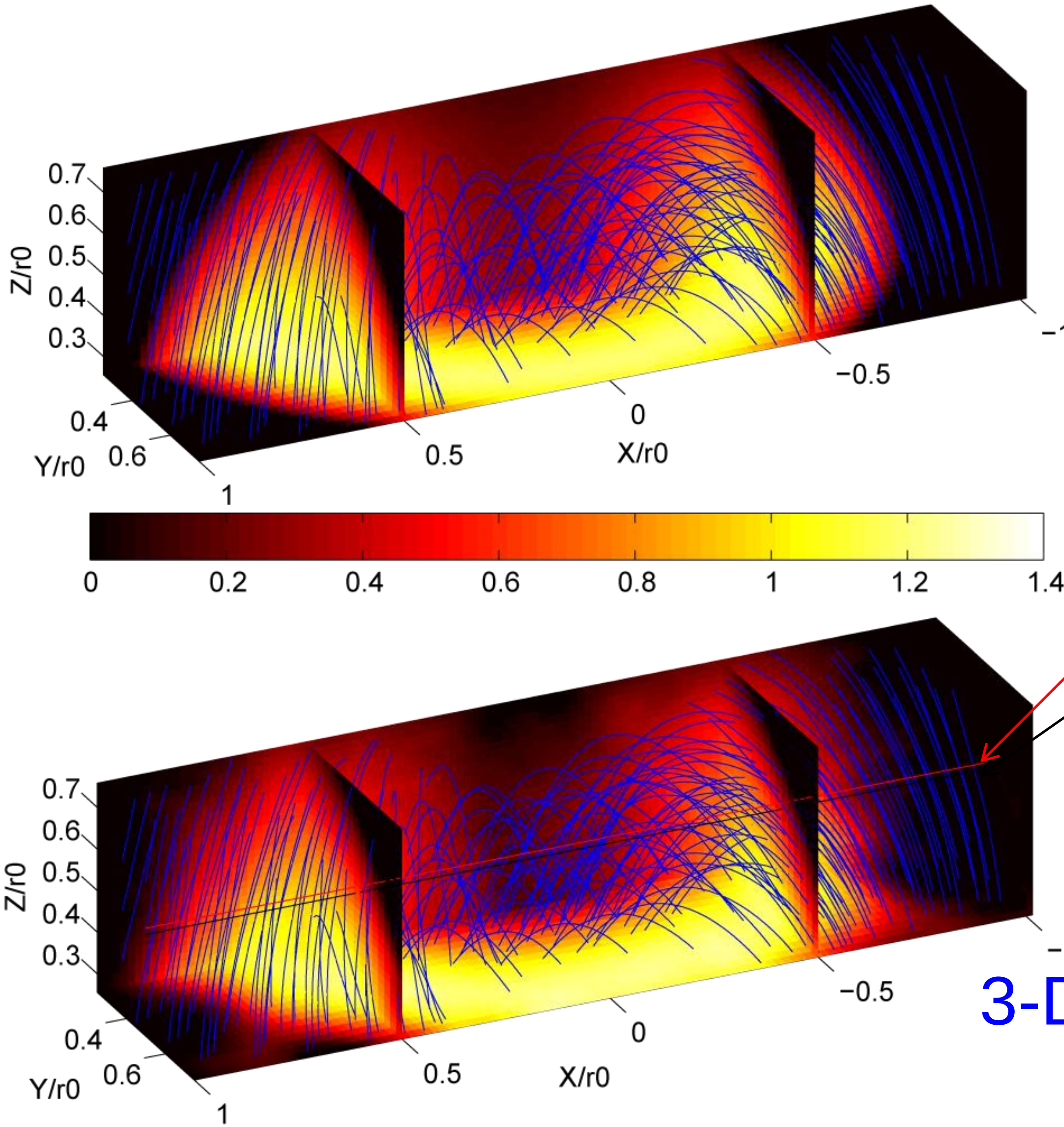
Structure-rest frame



Use of P makes computation more stable.

- Only 2 eqs. have terms divided by B_y.
- Total pressure P is less variable than p.

Benchmark test



Exact solution (axisymmetric Spheromak field)

Pressure

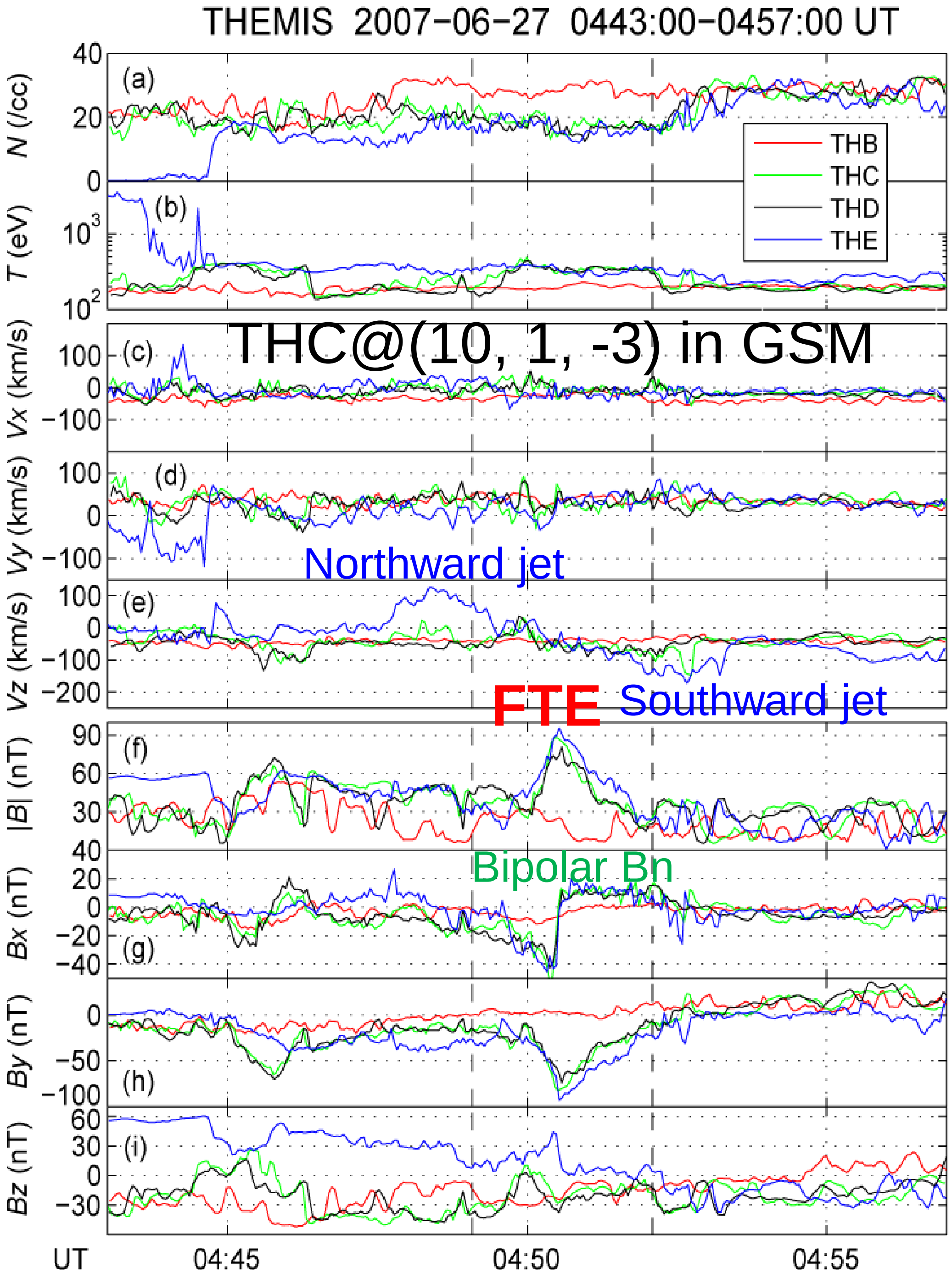
SC2: (Y, Z) = (0.5, 0.515)

SC1: (Y, Z) = (0.5, 0.5)

Separation: ~2% of the scale size

Reconstructed field

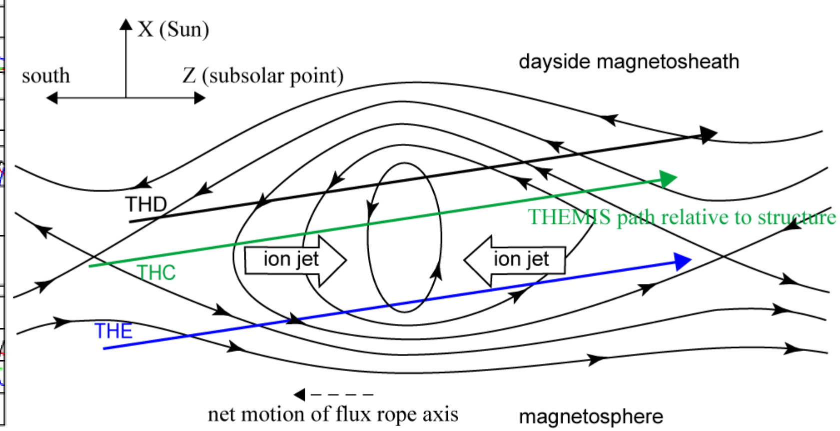
3-D field configuration & pressure

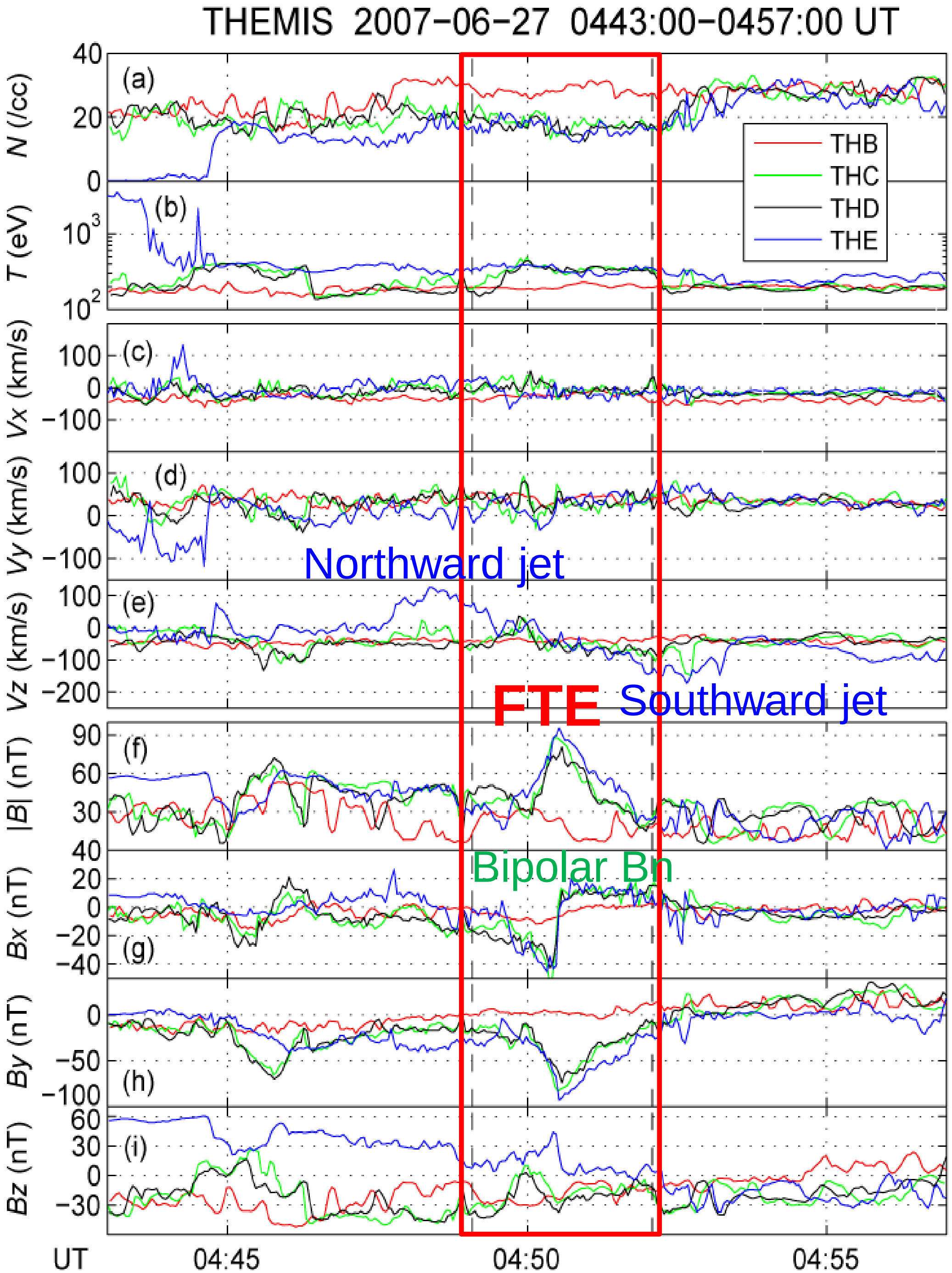


THEMIS observation of a flux transfer event (FTE) at the M'pause (2007-06-27)

- FTE was sandwiched between northward & southward jets.
- FTE moved southward.

$\mathbf{V}_{HT}$ = (-6, 24, -56) km/s.





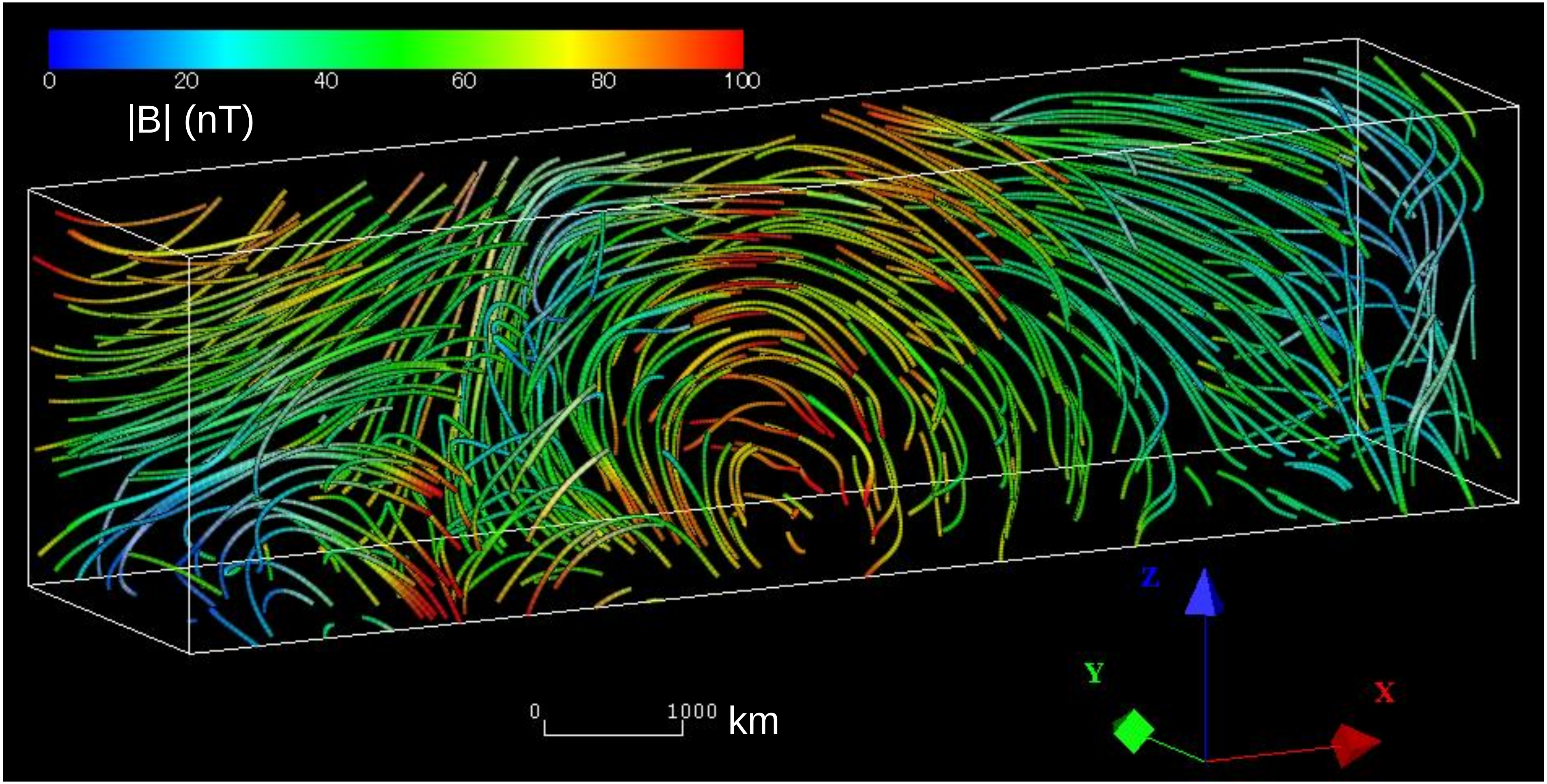
THEMIS observation of a flux transfer event (FTE) at the M'pause (2007-06-27)

- FTE was sandwiched between northward & southward jets.
- FTE moved southward.

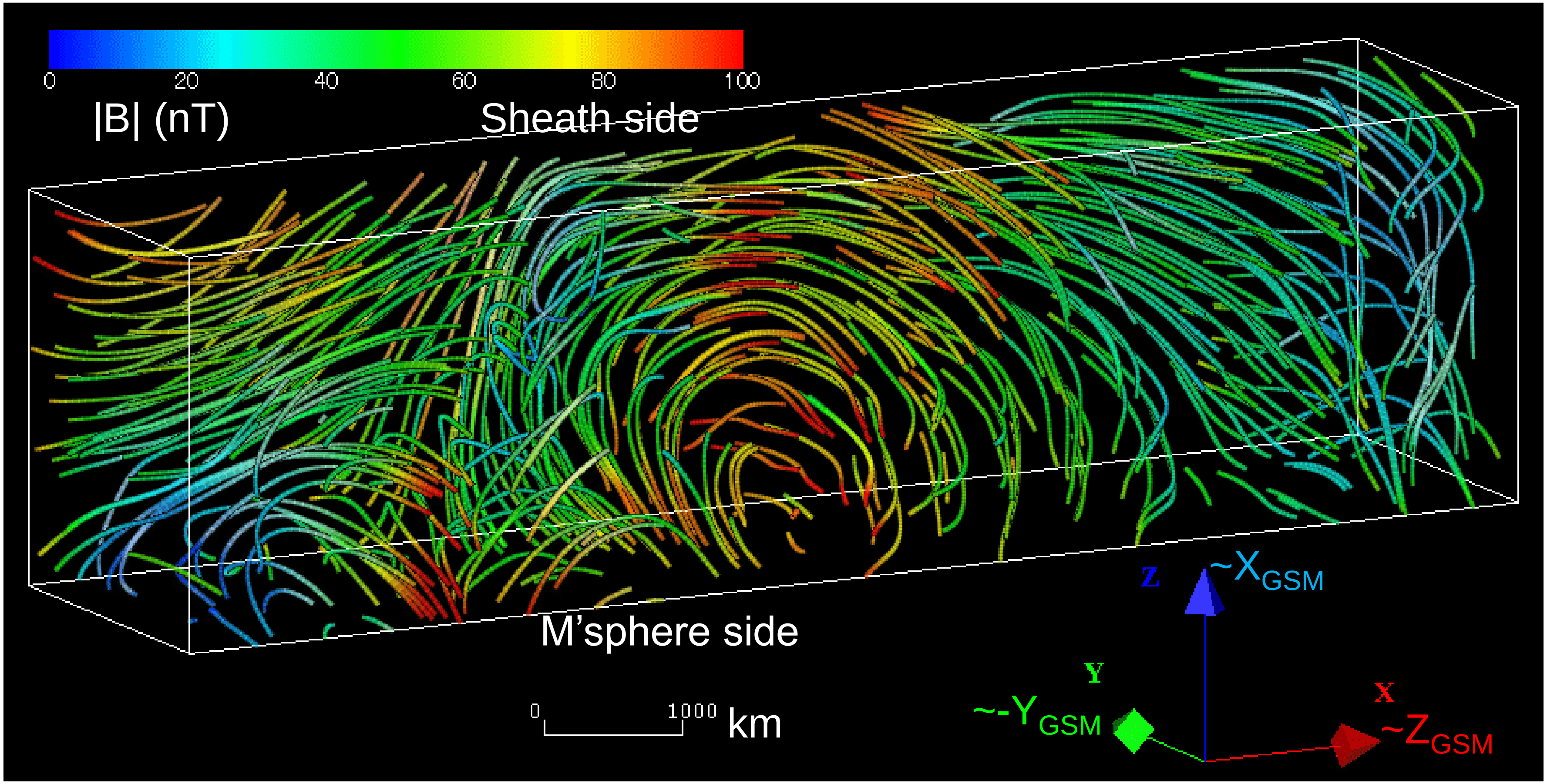
→ Generation by multiple X-line reconnection

- The method is applied to data from TH-C & -D, separated by ~390 km along MP normal.

3-D structure of a magnetic flux rope recovered from 2-SC (THEMIS) data

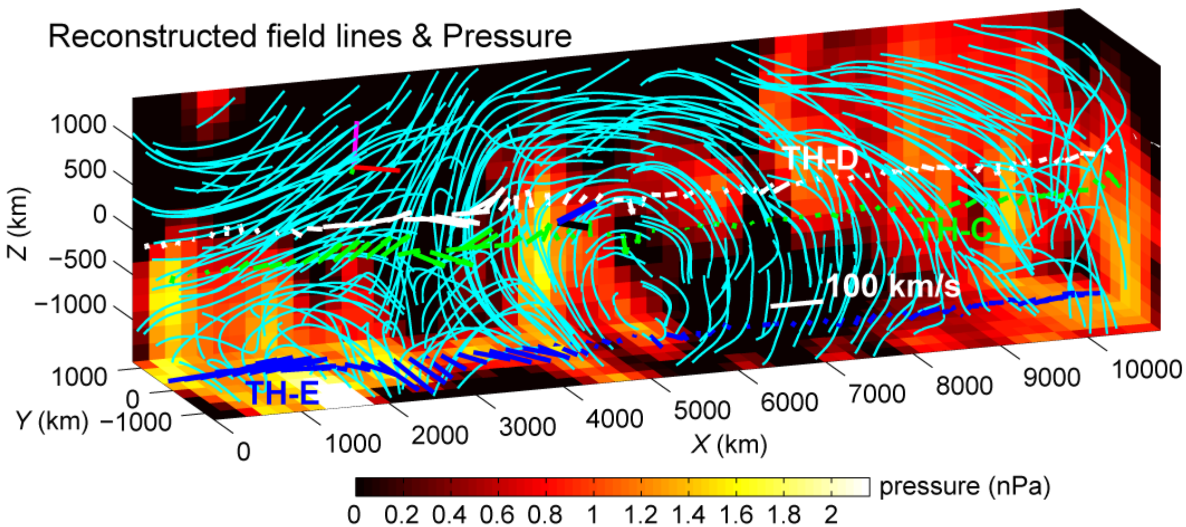


3-D structure of a magnetic flux rope recovered from 2-SC (THEMIS) data

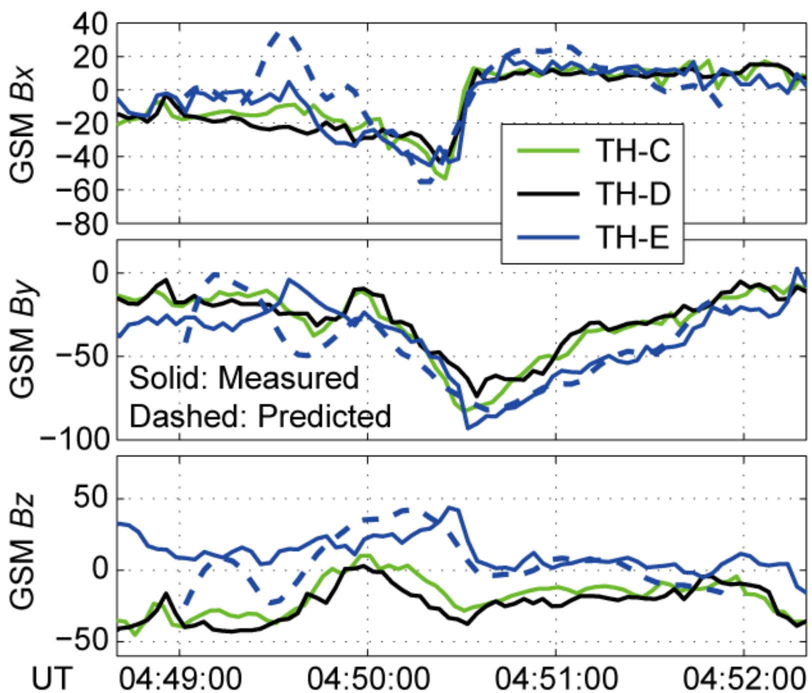


- Magnetic flux rope with diameter ~3000 km at the MP.

Predicted vs Observed *B* @TH-E



Sc traversed the structure (to the right) at ~61 km/s along the *X* axis.



- TH-E: closer to the Earth, & ~1300 km away from TH-C.
- Field variations (both polarity & intensity) at TH-E are well recovered.

Is the observed flux rope truly 3-D on 1000 km scale?

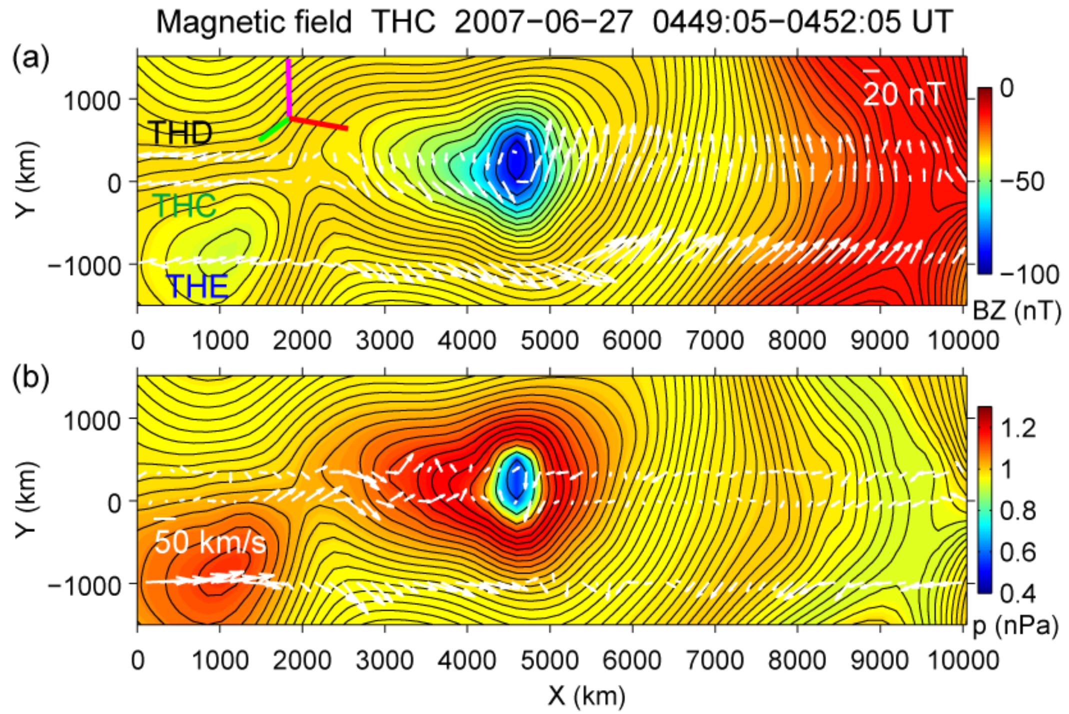
2-D Grad-Shafranov reconstruction

Assumptions:

- Magneto-hydrostatic
- 2-D ($\partial/\partial z \sim 0$)
  - In-plane *B* fields can be expressed by partial magnetic vector potential *A*.

Grad-Shafranov (GS) equation (e.g., Sturrock, 1994)

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu_0 \frac{dP_t}{dA},$$



$$A = A_z$$

$$P_t = (p + B_z^2 / 2\mu_0)$$

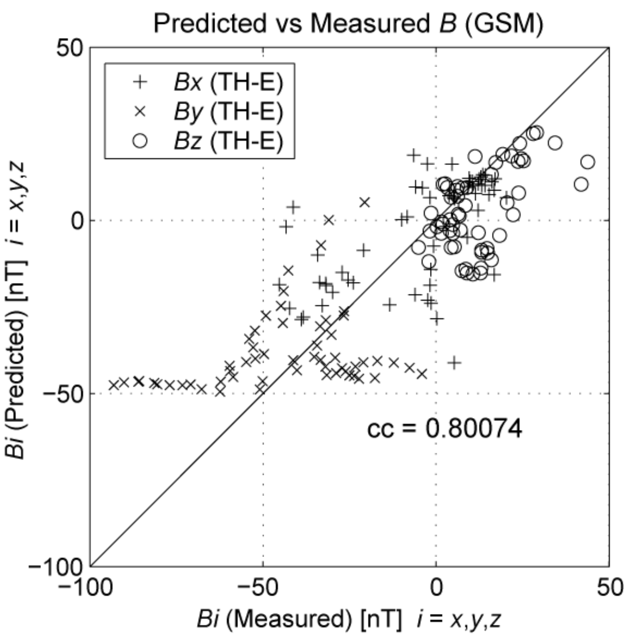
2-D maps of *B* & *p* can be recovered. (Hau & Sonnerup, JGR99)

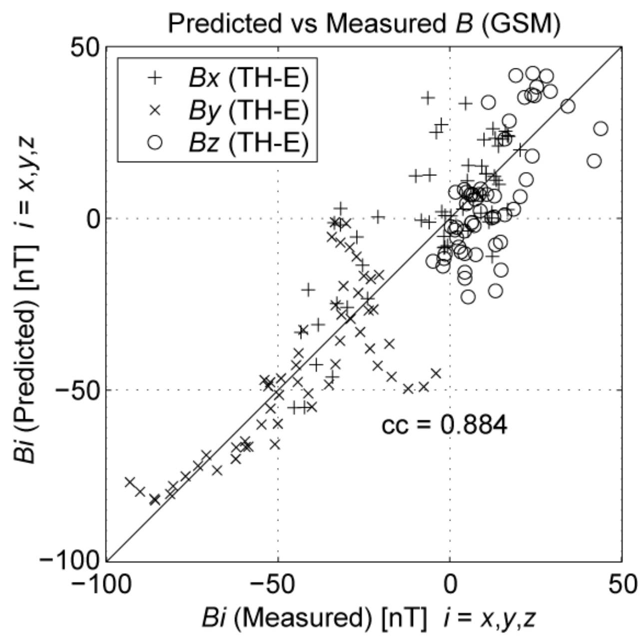
3-D (rather than 2-D) method better predicts *B*-field @TH-E

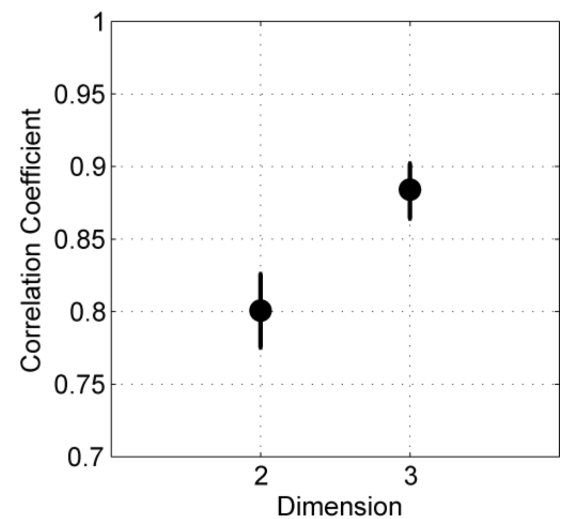
Correlation between predicted & measured fields @TH-E

2-D reconstruction (Grad-Shafranov)

3-D reconstruction (new method)



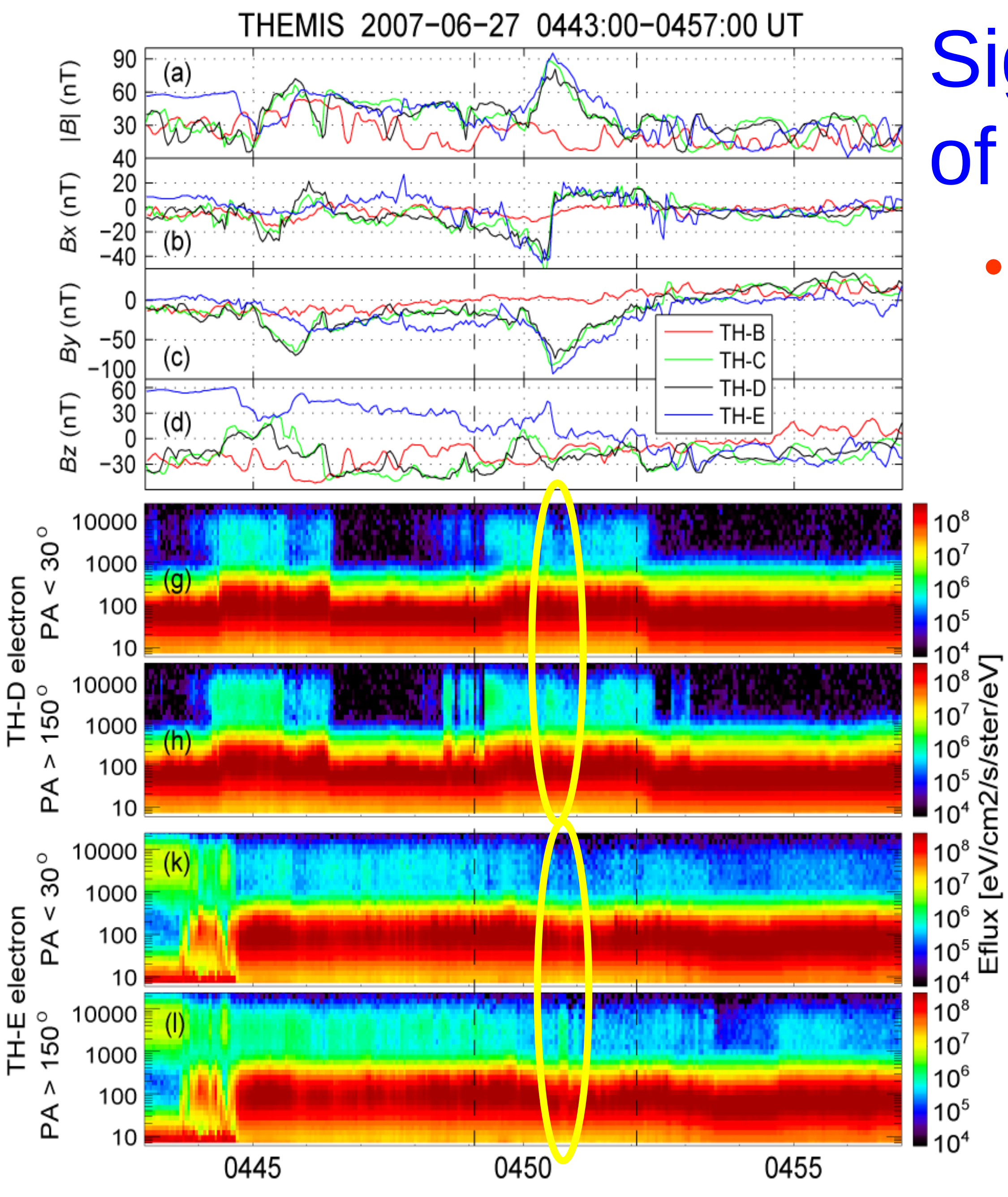




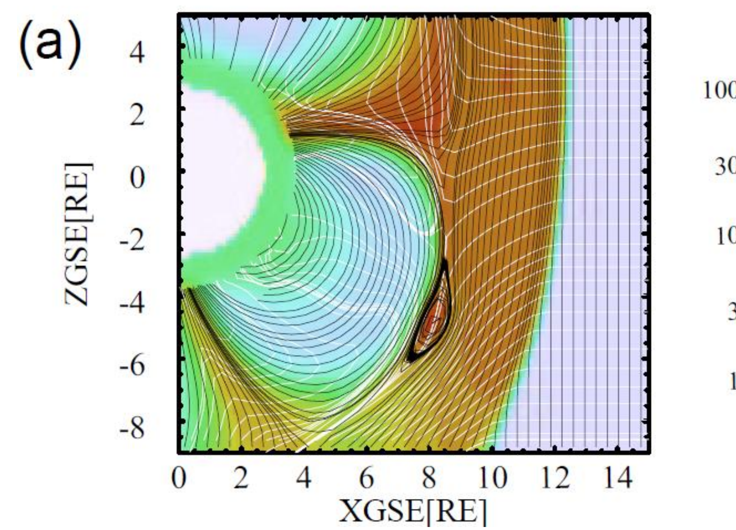
Error estimated by Bootstrap method

(Efron & Tibshirani, Stat. Sci.86; Kawano+, GRL95)

Sign of 3D topology of the flux rope

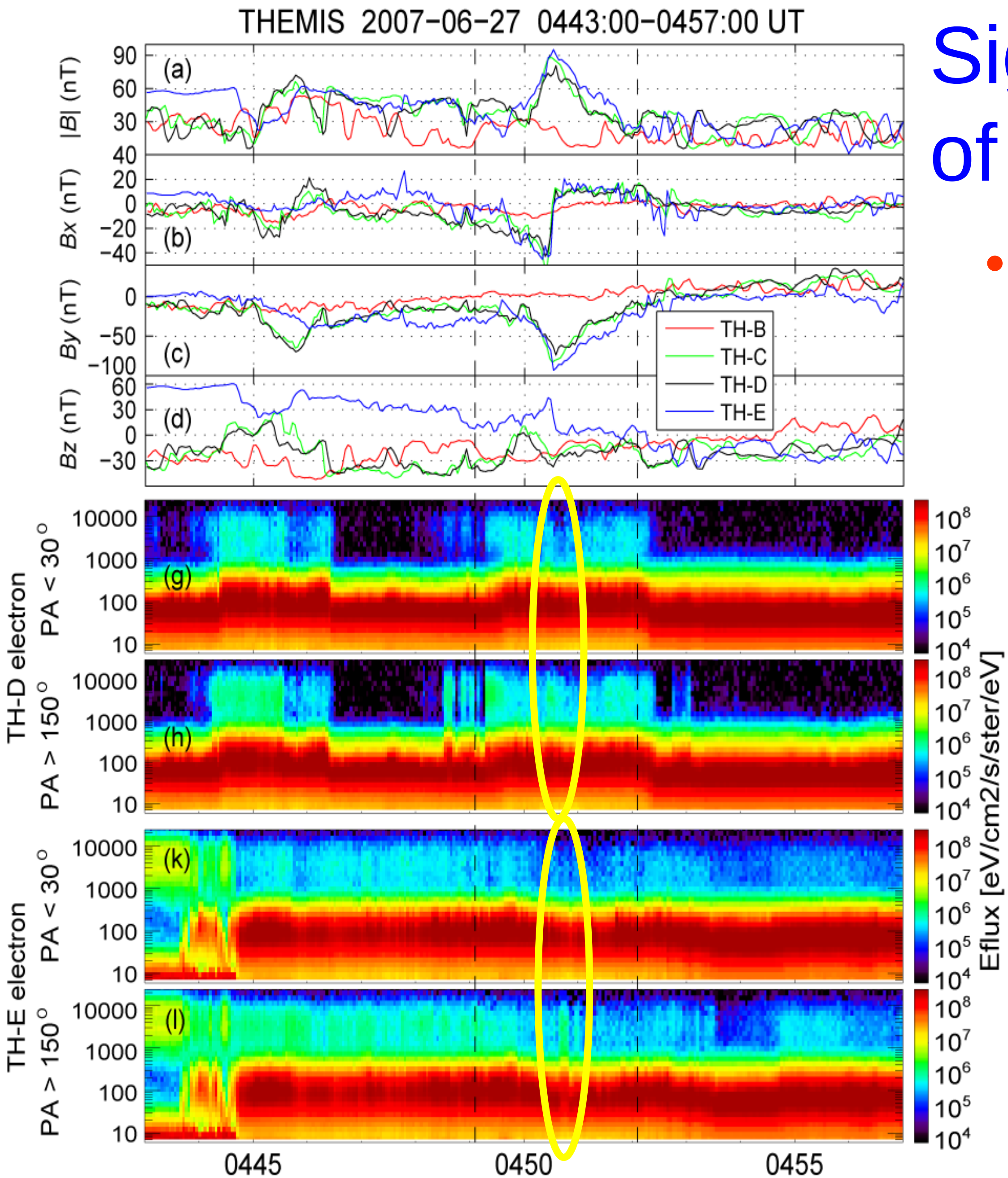


- Unbalanced electron field-aligned & anti-field-aligned fluxes at the flux rope center.
  - Field lines neither closed nor IMF-type.



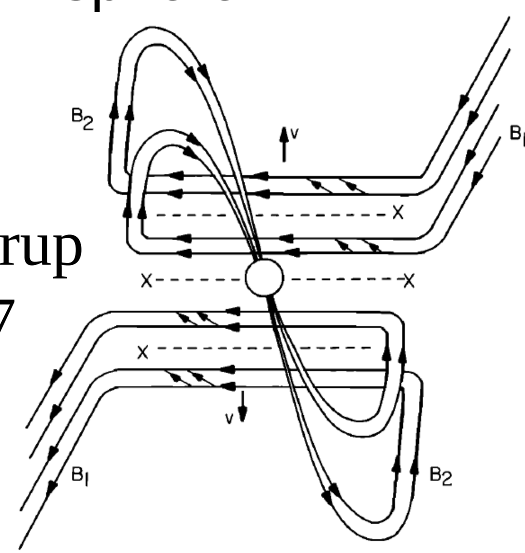
Raeder, AG06

Sign of 3D topology of the flux rope

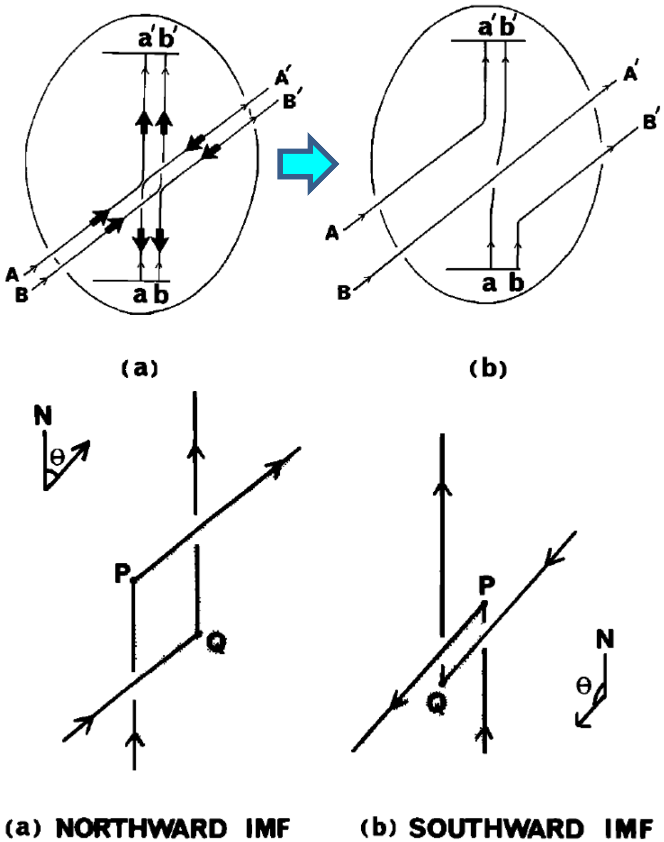


- Unbalanced electron field-aligned & anti-field-aligned fluxes at the flux rope center.
  - One end connected to the northern hemisphere.

Sonnerup JGR87



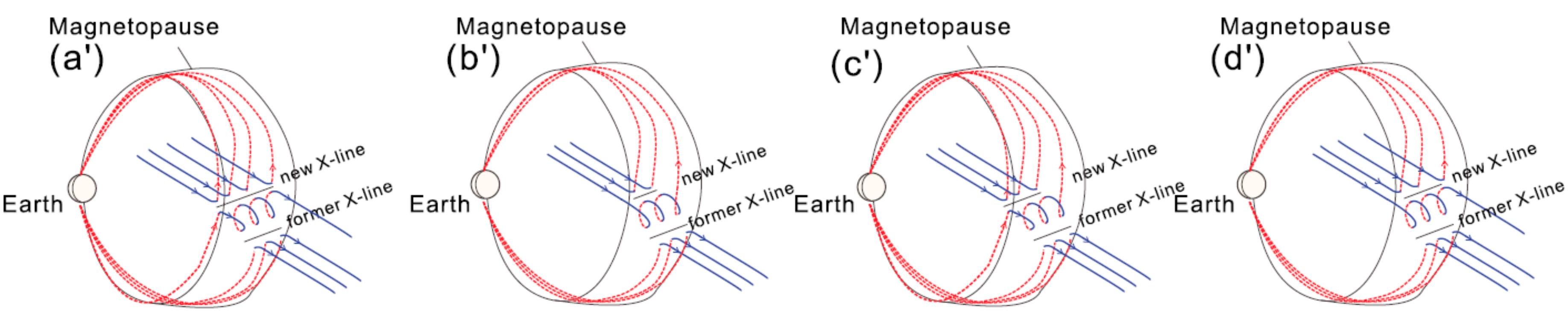
Potential impact of 3-D reconnection



- Field lines generated by 3-D multiple X-line reconnection can interact in a complex way, i.e., may be entangled, or intersect with each other.

Such interaction may modulate or suppress the efficiency of solar wind energy transfer.

Nishida (GRL89), Pu+ (GRL13)



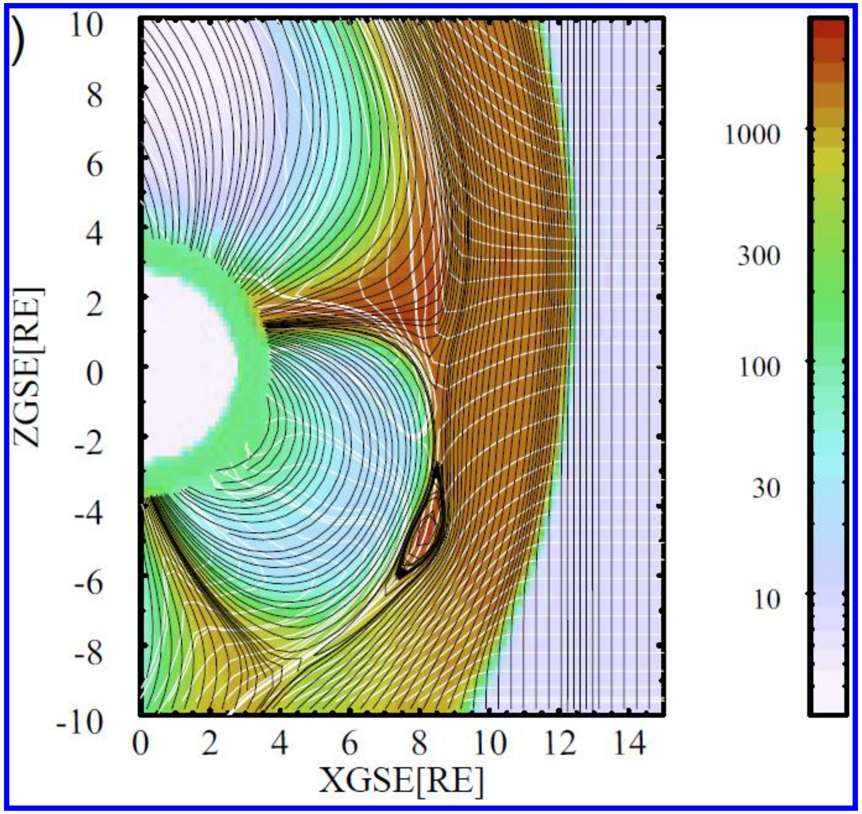
Summary

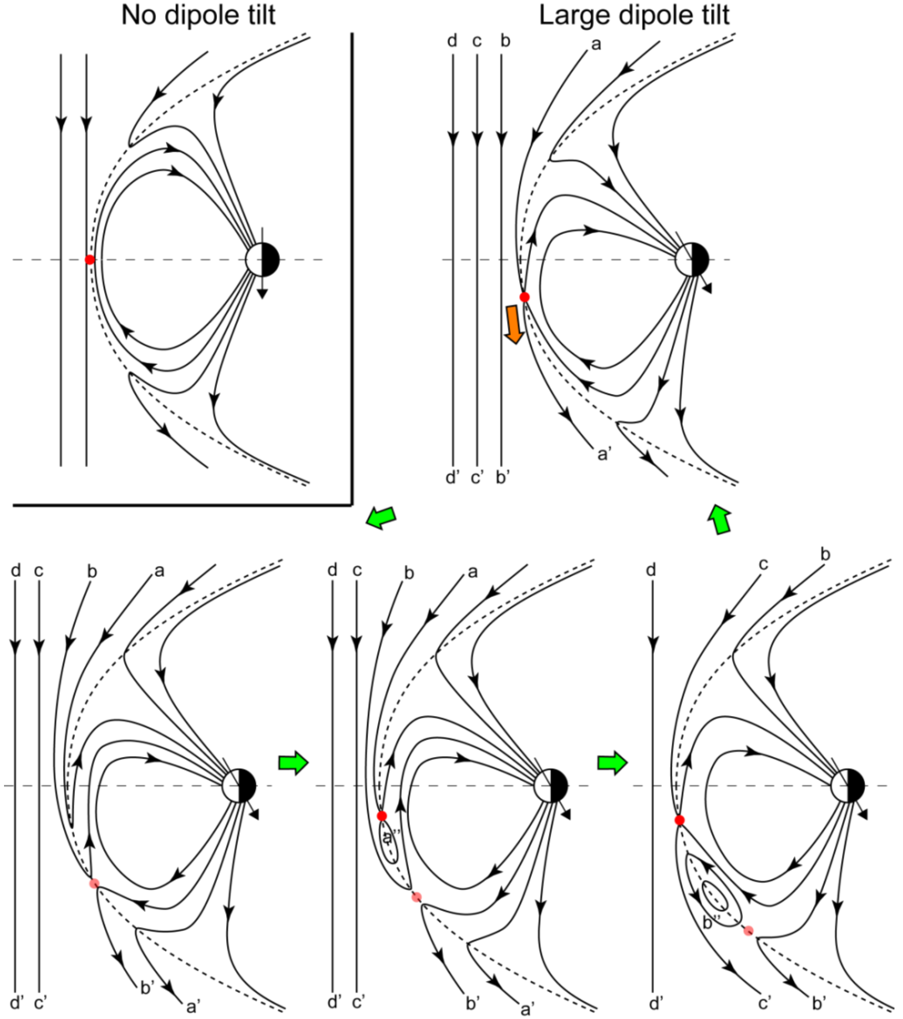
- First application (to an FTE) of a new data analysis method for the reconstruction of steady, 3-D, magneto-hydrostatic field structures from 2-Sc measurements.
- The reconstructed field of a magnetic flux rope observed at the subsolar magnetopause had a significant 3-D structure.
- Anisotropic electron pitch-angle distributions are consistent with a 3-D magnetic topology.

Complex interaction of IMF & geomag. fields may modulate/suppress SW energy transfer.

Potential impact of 3-D & multiple X-line reconnection

Hasegawa+ (GRL10)

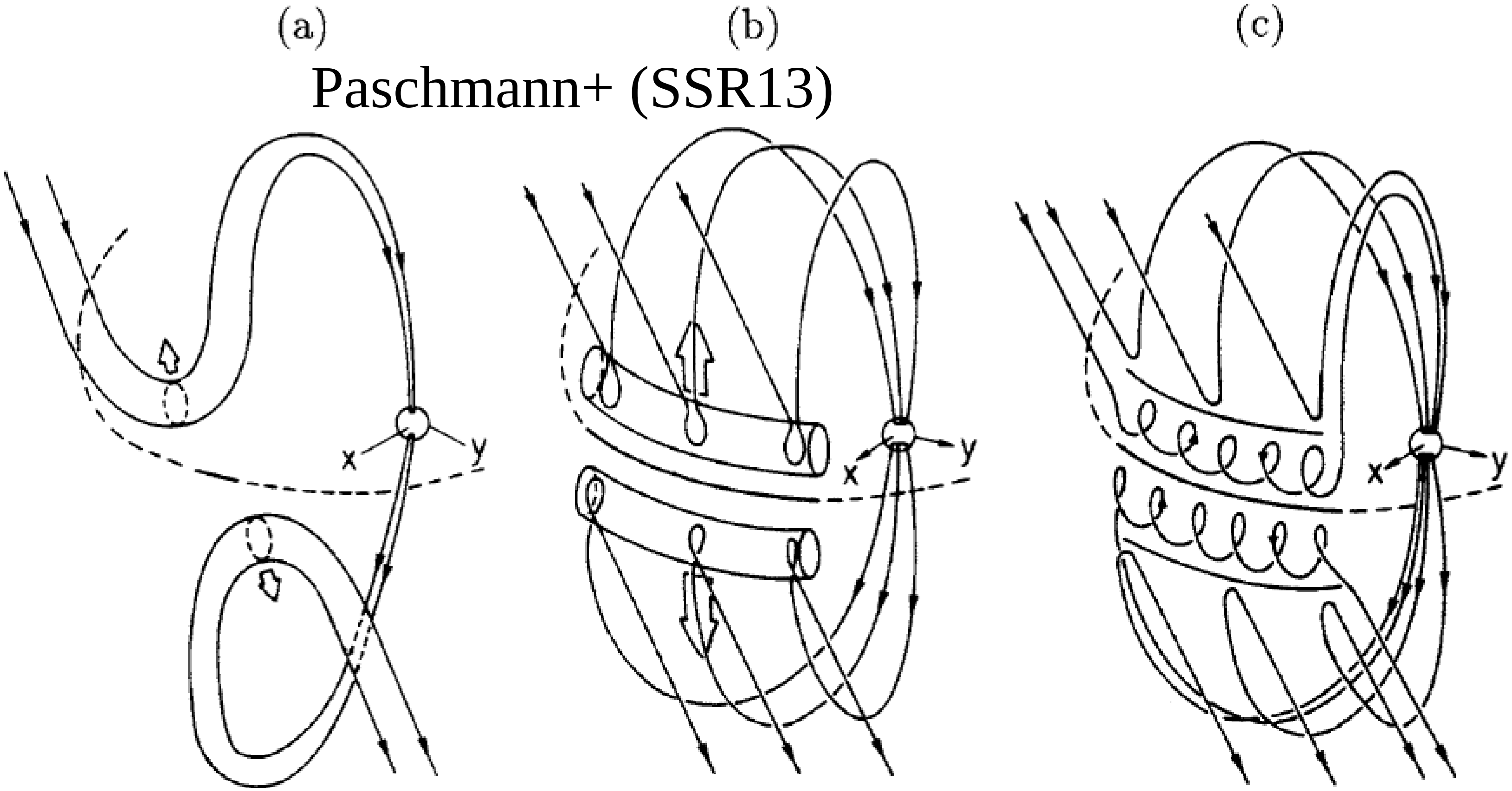




3-D global MHD simulation
with dipole tilt (Raeder, AG06)

Less efficient energy transfer for larger dipole tilt?

Prospects for 3-D reconstruction: FTE generation mechanism identification

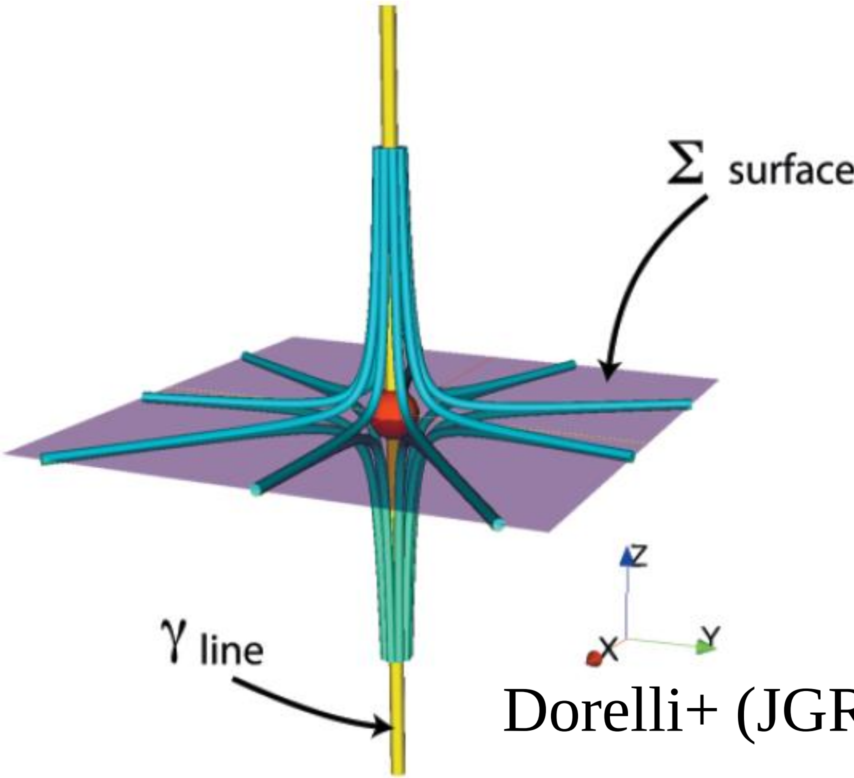


(a) Flux tube axis crosses the magnetopause surface.

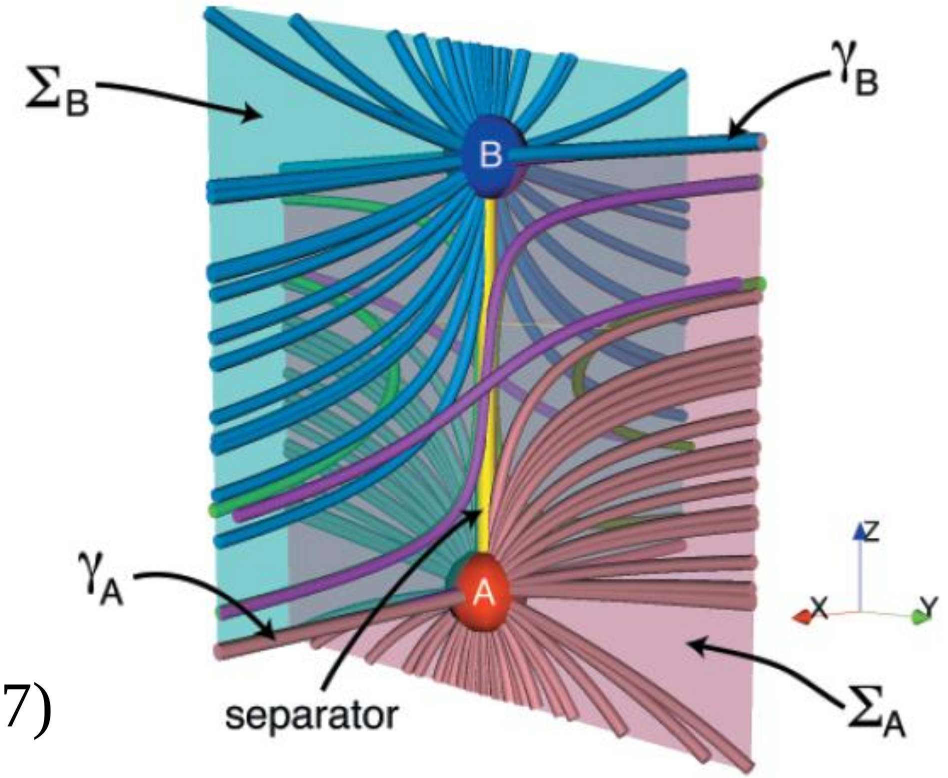
(b) No helical field lines (no full 360^{o} turn of field line).

(c) Helical field lines and multiple X-lines.

Prospects for 3-D reconstruction: Nulls & separator in 3-D reconnection

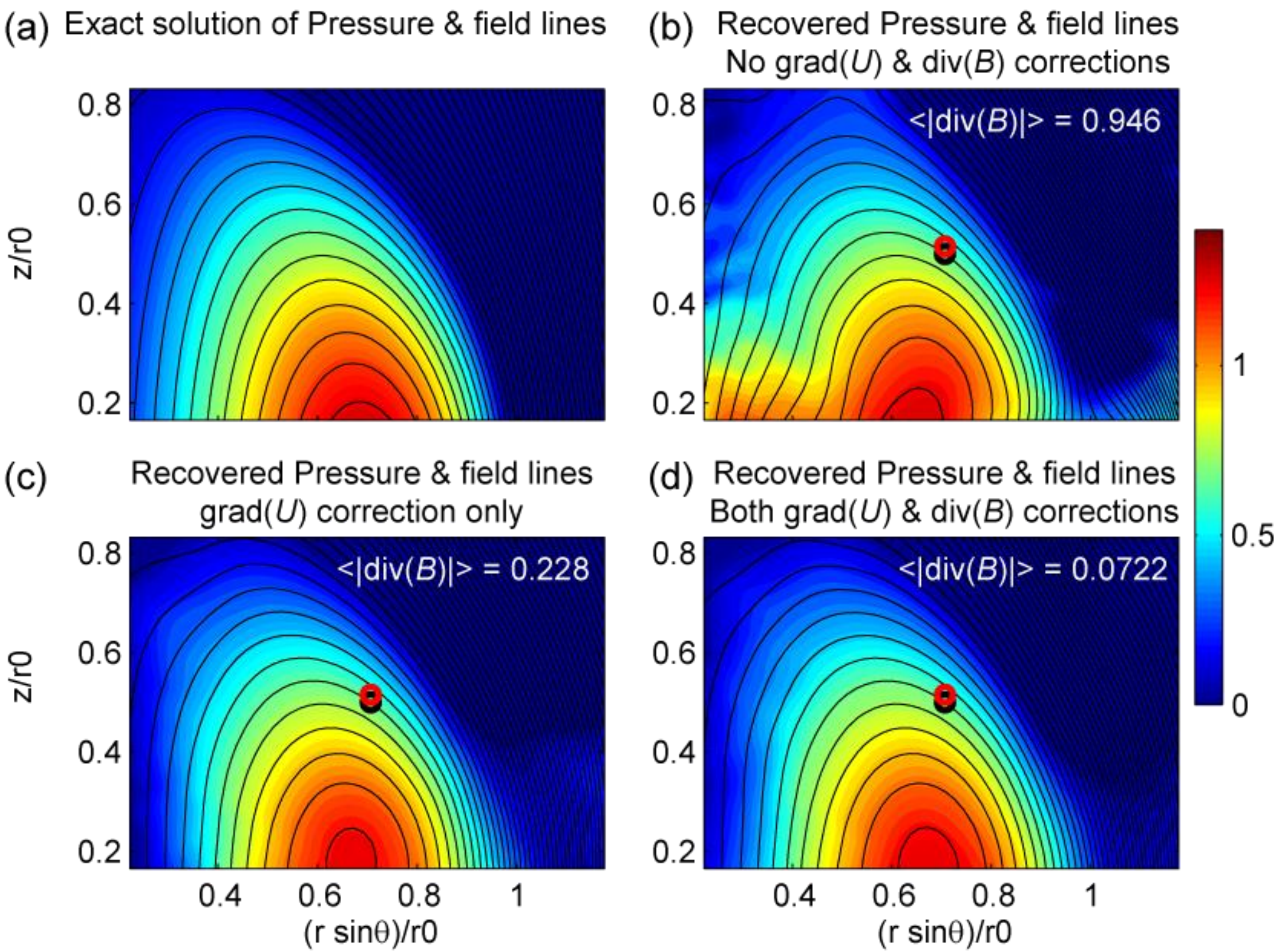


Dorelli+ (JGR07)



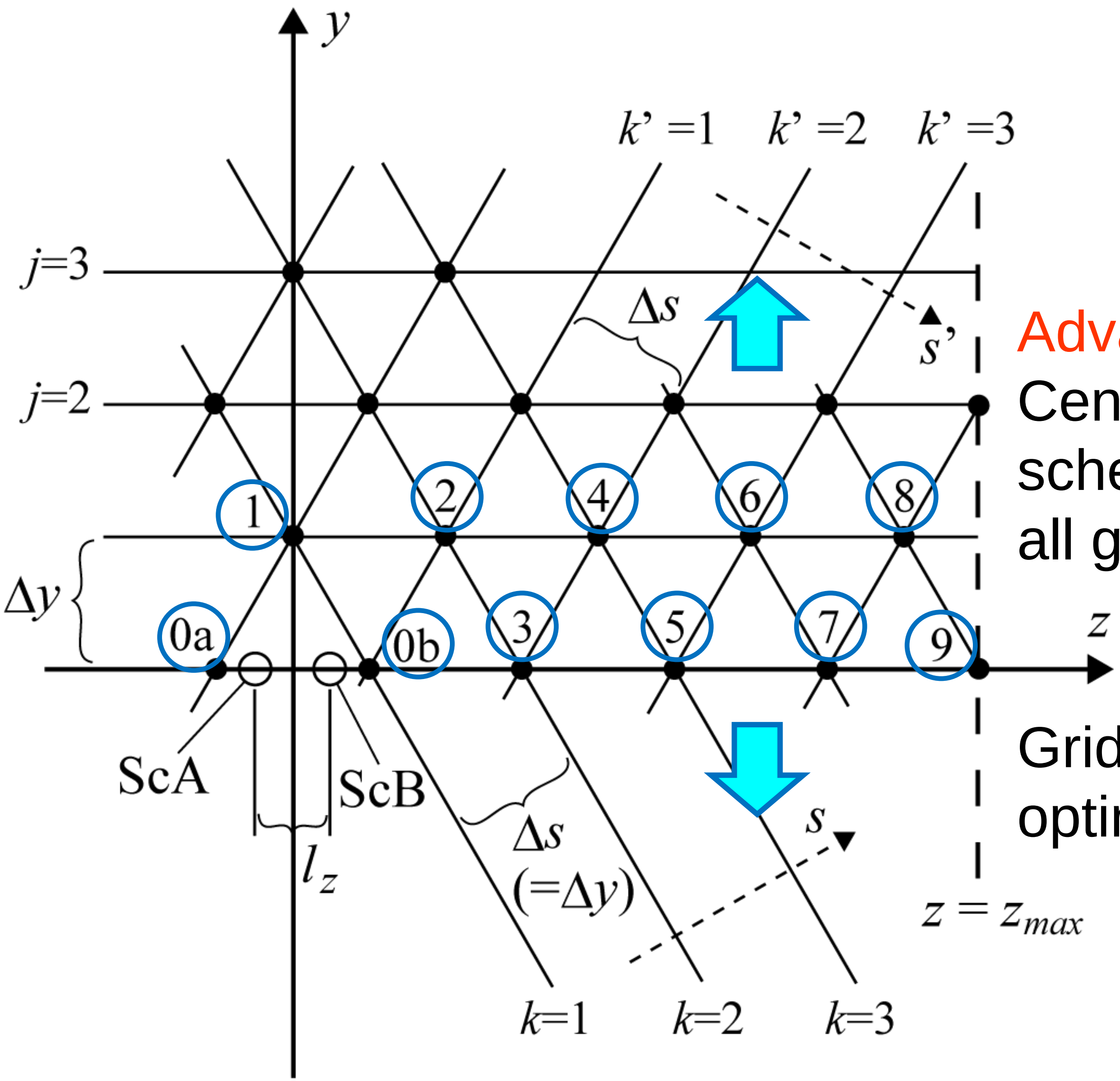
Key to identifying the topological boundary in 3-D field.
→ Estimation of the reconnection *E*-field integrated along the separator.

Quantitative comparison



→ : p should be const. along field lines.

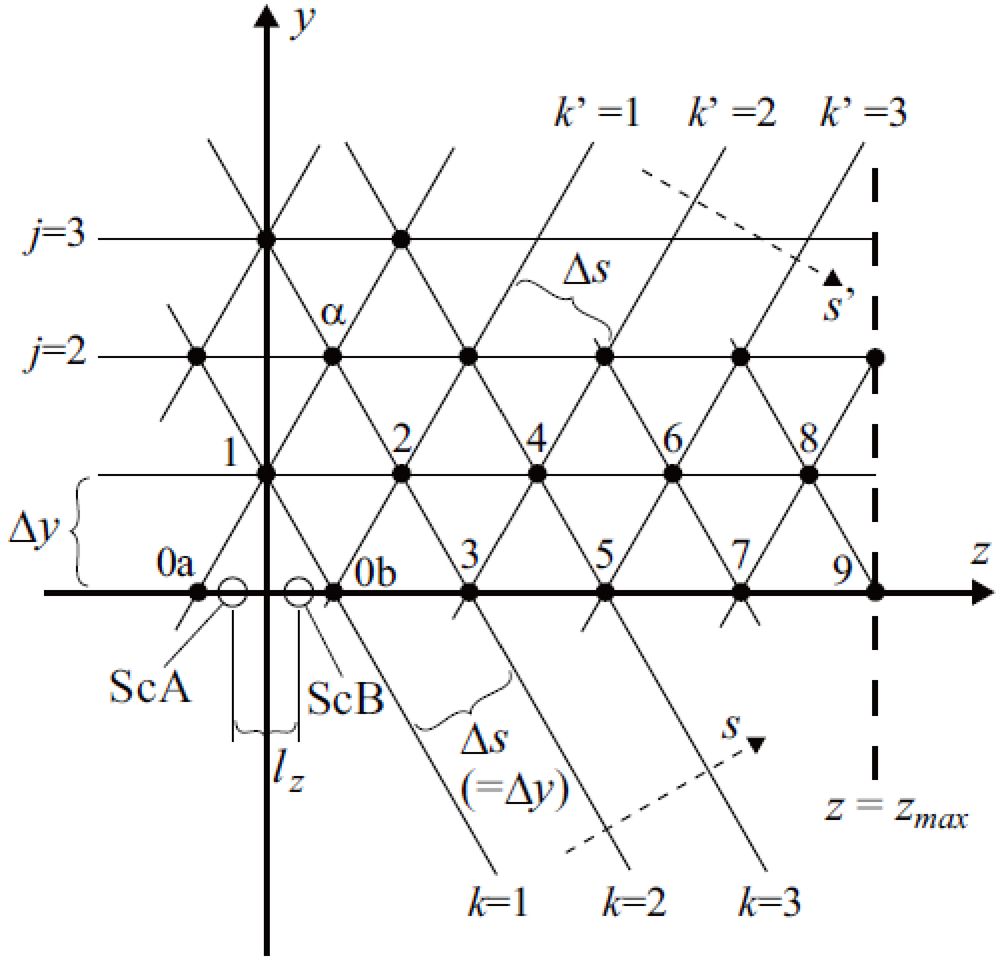
Actual computation grid



Advantage: Central difference scheme can be used at all grid points.

Grid cell size is optimized by trial & error.

How to reduce numerical errors: 1. Gradient correction



$$\left(\frac{\partial U}{\partial y}\right)_{j-1/2} = \frac{(U_1+U_2)/2-U_{0b}}{\Delta y},$$

where U_1, U_2, and U_{0b} are the values at Points 1, 2, and 0b, respectively, and simple linear interpolation is used to obtain the values at the midpoint. Substantial difference between $(\partial U/\partial y)_j$ and $(\partial U/\partial y)_{j-1/2}$ suggests that $(\partial U/\partial y)_j$ are not of acceptable accuracy likely because of the appearance of the singularities or use of the low-order scheme. Thus, we replace $(\partial U/\partial y)_j$ by $(\partial U/\partial y)_{j-1/2}$ if the absolute value of the difference $\left|(\partial U/\partial y)_j - (\partial U/\partial y)_{j-1/2}\right|$ exceeds a certain threshold value (set at 0.5 in the present study). Note that

How to reduce numerical errors: 2. div(B) correction

The value of $\nabla \cdot \boldsymbol{B}$ after the smoothing can be written as

$$\nabla \cdot \boldsymbol{B}_{j,\mathrm{old}} = \left(\frac{\partial B_x}{\partial x}\right)_j + \left(\frac{\partial B_y}{\partial y}\right)_{j,\mathrm{old}} + \left(\frac{\partial B_z}{\partial z}\right)_j = \beta_j, \tag{A2}$$

where β_j may be nonzero. We nudge the numerically estimated $\partial B_y / \partial y$ toward satisfying $\nabla \cdot \boldsymbol{B} = 0$ in the following way,

$$\left(\frac{\partial B_y}{\partial y}\right)_{j,\mathrm{new}} = \left(\frac{\partial B_y}{\partial y}\right)_{j,\mathrm{old}} - C\beta_j, \tag{A3}$$

where C is a nudging factor smaller than unity (set at 0.5 in the present study), and the subscripts "old" and "new" represent the values before and after the nudging correction, respectively. The corrected value $\left(\partial B_y / \partial y\right)_{j,\mathrm{new}}$ is used in the estimation of $\partial P / \partial y$ as well

Lessons learned from benchmarking

- Spacecraft separation should be ≤ 10% of the scale size of the structure.
- For numerical reasons,
  - Pressure (p) can be <0.
  -
  - p is not strictly constant along field lines.
  - It is difficult to well recover the regions of B_y or B_z reversal.

But overall, the 3-D reconstruction code works reasonably well.